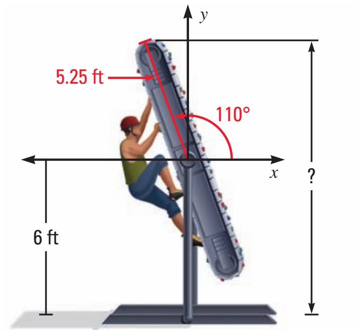
EXAMPLE 6 Model with a trigonometric function

ROCK CLIMBING A rock climber is using a rock climbing treadmill that is 10.5 feet long. The climber begins by lying horizontally on the treadmill, which is then rotated about its midpoint by 110° so that the rock climber is climbing towards the top. If the midpoint of the treadmill is 6 feet above the ground, how high above the ground is the top of the treadmill?



Solution

$\sin \theta = \dfrac{y}{r}$ — **Use definition of sine.**

$\sin 110° = \dfrac{y}{5.25}$ — **Substitute 110° for θ and $\frac{10.5}{2} = 5.25$ for r.**

$4.9 \approx y$ — **Solve for y.**

▶ The top of the treadmill is about $6 + 4.9 = 10.9$ feet above the ground.

✓ GUIDED PRACTICE for Examples 5 and 6

10. TRACK AND FIELD Estimate the horizontal distance traveled by a track and field long jumper who jumps at an angle of 20° and with an initial speed of 27 feet per second.

11. WHAT IF? In Example 6, how high is the top of the rock climbing treadmill if it is rotated 100° about its midpoint?

13.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 33, 37, and 39

SKILL PRACTICE

1. VOCABULARY Copy and complete: A(n) _?_ is an angle in standard position whose terminal side lies on an axis.

2. ★ WRITING Given an angle θ in Quadrant III, explain how you can use a reference angle to find $\cos \theta$.

EXAMPLE 1 on p. 866 for Exs. 3–11

USING A POINT **Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ.**

3. (8, 15)	**4.** (−9, 12)	**5.** (−7, −24)	**6.** (5, −12)
7. (2, −2)	**8.** (−6, 9)	**9.** (−3, −5)	**10.** $(5, -\sqrt{11})$

11. ★ MULTIPLE CHOICE Let (−7, −4) be a point on the terminal side of an angle θ in standard position. What is the value of $\tan \theta$?

Ⓐ $-\frac{7}{4}$ Ⓑ $-\frac{4}{7}$ Ⓒ $\frac{4}{7}$ Ⓓ $\frac{7}{4}$

EXAMPLE 2
on p. 867
for Exs. 12–15

QUADRANTAL ANGLES **Evaluate the six trigonometric functions of θ.**

12. $\theta = 0°$ 13. $\theta = \frac{\pi}{2}$ 14. $\theta = 540°$ 15. $\theta = \frac{7\pi}{2}$

EXAMPLE 3
on p. 868
for Exs. 16–23

FINDING REFERENCE ANGLES **Sketch the angle. Then find its reference angle.**

16. $-100°$ 17. $150°$ 18. $320°$ 19. $-370°$

20. $-\frac{5\pi}{6}$ 21. $\frac{8\pi}{3}$ 22. $\frac{15\pi}{4}$ 23. $-\frac{13\pi}{6}$

EXAMPLE 4
on p. 869
for Exs. 24–31

EVALUATING FUNCTIONS **Evaluate the function without using a calculator.**

24. $\sec 135°$ 25. $\tan 240°$ 26. $\sin(-150°)$ 27. $\csc(-420°)$

28. $\cos \frac{7\pi}{4}$ 29. $\cot\left(-\frac{8\pi}{3}\right)$ 30. $\tan\left(-\frac{3\pi}{4}\right)$ 31. $\sec \frac{11\pi}{6}$

32. **ERROR ANALYSIS** Let (4, 3) be a point on the terminal side of an angle θ in standard position. *Describe* and correct the error in finding $\tan \theta$.

$\tan \theta = \frac{x}{y} = \frac{4}{3}$ ✗

33. ★ **SHORT RESPONSE** Write $\tan \theta$ as the ratio of two other trigonometric functions. Use this ratio to explain why $\tan 90°$ is undefined but $\cot 90° = 0$.

34. **CHALLENGE** Five of the most famous numbers in mathematics — 0, 1, π, e, and i — are related by the simple equation $e^{\pi i} + 1 = 0$. Derive this equation using Euler's formula: $e^{a + bi} = e^a(\cos b + i \sin b)$.

PROBLEM SOLVING

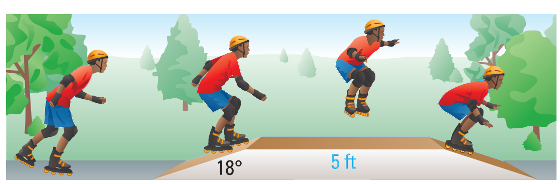
EXAMPLE 5
on p. 869
for Exs. 35–36

In Exercises 35 and 36, use the formula in Example 5 on page 869.

35. **FOOTBALL** You and a friend each kick a football with an initial speed of 49 feet per second. Your kick is projected at an angle of 45° and your friend's kick is projected at an angle of 60°. About how much farther will your football travel than your friend's football?

@HomeTutor for problem solving help at classzone.com

36. **IN-LINE SKATING** At what speed must the in-line skater launch himself off the ramp in order to land on the other side of the ramp?

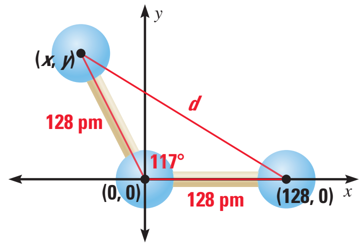


@HomeTutor for problem solving help at classzone.com

EXAMPLE 6
on p. 870
for Exs. 37–38

37. ★ **SHORT RESPONSE** A Ferris wheel has a radius of 75 feet. You board a car at the bottom of the Ferris wheel, which is 10 feet above the ground, and rotate 255° counterclockwise before the ride temporarily stops. How high above the ground are you when the ride stops? If the radius of the Ferris wheel is doubled, is your height above the ground doubled? *Explain*.

38. MULTI-STEP PROBLEM When two atoms in a molecule are bonded to a common atom, chemists are interested in both the bond angle and the lengths of the bonds. An ozone molecule (O_3) is made up of two oxygen atoms bonded to a third oxygen atom, as shown.



a. In the diagram, coordinates are given in picometers (pm). (*Note:* 1 pm $= 10^{-12}$ m.) Find the coordinates (x, y) of the center of the oxygen atom in Quadrant II.

b. Find the distance d (in picometers) between the centers of the two unbonded oxygen atoms.

39. ★ EXTENDED RESPONSE A sprinkler at ground level is used to water a garden. The water leaving the sprinkler has an initial speed of 25 feet per second.

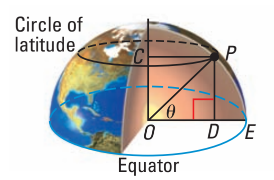
a. Calculate Copy the table below. Use the formula in Example 5 on page 869 to complete the table.

Angle of sprinkler, θ	25°	30°	35°	40°	45°	50°	55°	60°	65°
Horizontal distance water travels, d	?	?	?	?	?	?	?	?	?

b. Interpret What value of θ appears to maximize the horizontal distance traveled by the water? Use the formula for horizontal distance traveled and the unit circle to explain why your answer makes sense.

c. Compare *Compare* the horizontal distance traveled by the water when $\theta = (45 - k)°$ with the distance when $\theta = (45 + k)°$.

40. CHALLENGE The latitude of a point on Earth is the degree measure of the shortest arc from that point to the equator. For example, the latitude of point P in the diagram equals the degree measure of arc PE. At what latitude θ is the circumference of the circle of latitude at P half the distance around the equator?



MIXED REVIEW

PREVIEW
Prepare for Lesson 13.4 in Exs. 41–46.

Graph the function *f*. Then use the graph to determine whether the inverse of *f* is a function. *(p. 438)*

41. $f(x) = 5x + 2$ **42.** $f(x) = -x + 7$ **43.** $f(x) = x^2 + 5$

44. $f(x) = 4x^2, x \geq 0$ **45.** $f(x) = 0.25x^2$ **46.** $f(x) = |x - 7|$

Find the range and standard deviation of the data set. *(p. 744)*

47. 3, 5, 2, 3, 7, 11, 8, 4 **48.** 18, 12, 15, 9, 13, 7, 4, 17

49. 5.9, 8.2, 3.7, 6.1, 2.9, 1.8, 5.7 **50.** 54, 60, 57, 53, 59, 51, 56, 62

Find the sum of the series.

51. $\sum_{i=1}^{15} (3i + 2)$ *(p. 802)* **52.** $\sum_{i=1}^{18} (4i + 1)$ *(p. 802)* **53.** $\sum_{i=1}^{24} (17 - 2i)$ *(p. 802)*

54. $\sum_{i=1}^{5} 2(3)^{i-1}$ *(p. 810)* **55.** $\sum_{i=1}^{7} \frac{1}{4}\left(\frac{3}{2}\right)^{i-1}$ *(p. 810)* **56.** $\sum_{i=1}^{\infty} 8\left(\frac{1}{2}\right)^{i-1}$ *(p. 820)*

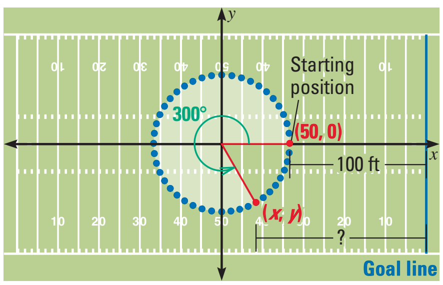
EXTRA PRACTICE for Lesson 13.3, p. 1022 **ONLINE QUIZ** at classzone.com

MIXED REVIEW *of Problem Solving*

STATE TEST PRACTICE
classzone.com

Lessons 13.1–13.3

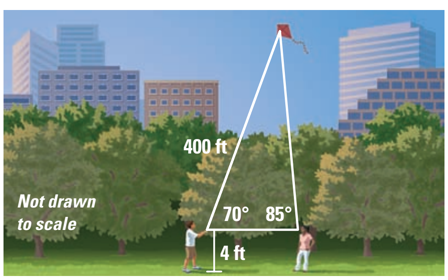
1. **MULTI-STEP PROBLEM** Your school's marching band is performing at halftime during a football game. In the last formation, the band members form a circle 100 feet wide in the center of the field. You start at a point on the circle 100 feet from the goal line, march 300° around the circle, and then walk toward the goal line to exit the field.



   a. How far from the goal line are you at the point where you leave the circle?

   b. How far do you march around the circle?

2. **MULTI-STEP PROBLEM** You are flying a kite at an angle of 70°. You have let out a total of 400 feet of string and are holding the reel steady 4 feet above the ground.



   a. How high above the ground is the kite?

   b. A friend watching the kite estimates that the angle of elevation to the kite is 85°. How far from your friend are you standing?

3. **GRIDDED ANSWER** What is the reference angle, in degrees, for the angle $\theta = 560°$?

4. **OPEN-ENDED** What is the measure, in degrees, of an angle for which the secant is positive and the cotangent is negative?

5. **SHORT RESPONSE** The top of the Space Needle in Seattle, Washington, is a revolving, circular restaurant. The restaurant has a radius of 47.25 feet and makes one complete revolution in about an hour. You have dinner at a window table from 7:00 P.M. to 8:55 P.M.

   a. How many feet do you revolve?

   b. Do diners seated 5 feet away from the windows revolve the same distance? *Explain*.

6. **MULTI-STEP PROBLEM** You are standing 100 meters from the main entrance of the Sears Tower in Chicago, Illinois. You estimate that the angle of elevation to the top of the skyscraper is 77°.

   a. What is the approximate height h of the Sears Tower?

   b. Suppose one of your friends is at the top of the Sears Tower. What is the straight-line distance d between you and your friend?

7. **EXTENDED RESPONSE** A pizza shop offers two choices for individual pizza slices, as shown.

   a. Find the area of each slice of pizza.

   b. Which slice is the better deal? *Explain* your reasoning.

   c. How could you change the price of the 7 inch slice so that neither slice offers a better deal than the other?